(30 min.) You are the design engineer in charge of selecting the coolant for a whole new reactor design. You may use either water or sodium, but the goal of your selection process is to achieve maximum heat transfer capability with minimum pumping power requirements. You decide to compute the dimensionless ratio \( \frac{q}{w} \), where \( q \) is the heat transfer rate from the cladding to the coolant (MW) and \( w \) is the pumping power (MW). You analyze the generic fuel pin-coolant channel shown below.

\[ A_c = \text{channel flow area} \]
\[ P \]
\[ \text{wetted perimeter} = \pi d \]
\[ L \]
\[ V = \text{coolant velocity} \]

Where

\[ \dot{q} = h_{DB}PL(T_C - T_{bulk}) \] — heat transfer rate to coolant

\( h_{DB} \) = single phase heat transfer coefficient (Dittus-Boelter)

\( T_C \) = core average cladding temperature

\( T_{Bulk} \) = core average coolant temperature
\[ \dot{w} = AC \frac{V}{D_c} \rho \frac{V^2}{2} \text{ Pumping Power} \]

\[ D_c = \text{channel equivalent hydraulic diameter} \]

\[ f = \text{friction factor} = 0.184 \text{Re}^{-0.2} \]

\[ \text{Nu} = 0.023 \text{Re}^{0.8} \text{Pr}^{0.3} \text{ (DB correlation, applicable to water and sodium in this case)} \]

You may find the following dimensionless numbers useful:

Nusselt number \[= \frac{h D_c}{k}\]

Reynolds number \[= \frac{\rho V D_c}{\mu}\]

Prandtl number \[= \frac{\mu C_p}{k}\]

Mach number \[= \frac{V}{\sqrt{C_p C_v RT}}\]

Stanton number \[= \frac{h}{\rho V C_p}\]

(A) (50%) Form the dimensionless ratio \(\frac{\dot{q}}{\dot{w}}\) and simplify as possible.

(B) (30%) Form the dimensionless ratio \(\frac{\dot{q}}{\dot{w}}\) \text{ water} \ and simplify as possible.

C (20%) For the properties listed below, evaluate your answer from Part B and explain the coolant you would select.

<table>
<thead>
<tr>
<th></th>
<th>(C_p) (BTU/lbm-(^\circ)F)</th>
<th>(k) (BTU/hr-ft-(^\circ)F)</th>
<th>(\mu) (lbm/ft-hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>water</td>
<td>1.51</td>
<td>0.292</td>
<td>0.120</td>
</tr>
<tr>
<td>sodium</td>
<td>0.312</td>
<td>43.8</td>
<td>0.835</td>
</tr>
</tbody>
</table>
2. (30 min.) Assume that an existing GE BWR power plant is operating under normal steady-state conditions, producing 100% of its full power. Following operation of considerable length of time at this power level, a 50% of full power step load decrease without steam bypass is introduced to the plant.

(85%) A. Sketch the transient responses of the following variables starting from the time the step load change is introduced, until a new steady-state is achieved:

(10%) (1) Steam pressure,
(10%) (2) Reactor downcomer water level,
(10%) (3) Core coolant inlet temperature,
(10%) (4) Core coolant outlet temperature,
(15%) (5) Average core coolant temperature,
(15%) (6) Neutronic power,
(15%) (7) Total core reactivity.

(15%) B. Assuming that no external reactivity is added to or removed from the core, i.e. no control rod movement has taken place, would a change in the demanded load always result in a new steady-state operating condition? Explain.

3. (15 min.) A steel reactor pressure vessel can be approximated as a slab. The thickness of the vessel is $x_0$ cm. Due to deposition of gammas from the core, there is an internal heat source in the pressure vessel given as $q''''(x) = q_i'''' e^{\mu x}$, where $q_i''''$ is the volumetric heat source density in W/cm$^3$ at $x=0$. The heat flux on the pressure vessel at $x=0$ is $q''_{x=0}$ W/cm$^2$. The outer temperature of the pressure vessel at $x = x_0$ is $t_o$ °C, and the thermal conductivity of the steel pressure vessel is $k$ W/cm °C. With the data given below, find the maximum temperature in the pressure vessel.

Slab thickness, $x_0 = 20$ cm

Volumetric heat source density at $x = 0$, $q_i'''' = 0.01$ W/cm$^3$

Attenuation coefficient in the pressure vessel, $\mu = 0.025$ cm$^{-1}$

Heat flux at $x = 0$, $q''_{x=0} = 0.275$ W/cm$^2$

Outer temperature of the pressure vessel at $x = x_0$ is $t_o = 230$°C

Thermal conductivity of the steel, $k = 0.18$ W/cm °C
4. (20 min.) Consider an experiment in which air and water flow in a tube of height of 1 meter and a diameter of 0.2 meter. The air passes through a porous plate at the bottom of the tube and water is pumped in from the side (the water is recycled by an overflow line as shown in the figure). Consider the following two cases:

**Case 1.** \( j_g = 0.1 \text{ m/s} \) and \( j_f = 0.1 \text{ m/s} \) for air and water respectively, at isothermal conditions at 10 atm.

**Case 2.** \( j_g = 0.2 \text{ m/s} \) and \( j_f = 0.0 \text{ m/s} \) for air and water respectively, at isothermal conditions at 1 atm.

**Find:**
1) the flow regime for each case
2) the void fraction using the drift flux model. List and justify any assumptions you feel you need to make.

**Properties:**

- water density = 1000 kg/m³
- water viscosity = 0.001 kg/m·s
- surface tension = 0.07 N/m
- gas density = 1 kg/m³
- gas viscosity = 0.0001 kg/m·s

**Hint:**

\[
\bar{u}_g = c_o \langle j \rangle + \bar{u}_{gj}
\]

where:
- \( c_o = 1.13 \)
- \( \bar{u}_{gj} = 1.41 \left( \frac{\sigma_g (\rho_f - \rho_g)}{\rho_f^2} \right)^{1/4} \)
Flow pattern map for vertical flow (Hewitt and Roberts 1969).
5. (25 minutes) Systems A and B provide emergency cooling water on demand at a nuclear power plant. Both systems share the same pump (P1) but employ separate, independent valves (VA and VB). Because systems A and B share a common component, P1, they are not independent. The pump and valves are sketched below; the remaining components of systems A and B may be ignored for the purposes of this problem.

A probabilistic risk assessment has developed an event tree for this plant, and the segment of that event tree involving systems A and B is shown below:

<table>
<thead>
<tr>
<th>System A</th>
<th>System B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Sys A succeeds and sys B succeeds.</td>
<td></td>
</tr>
<tr>
<td>2. Sys A succeeds and sys B fails.</td>
<td></td>
</tr>
<tr>
<td>3. Sys A fails and sys B succeeds.</td>
<td></td>
</tr>
<tr>
<td>4. Sys A fails and sys B fails.</td>
<td></td>
</tr>
</tbody>
</table>

For system A to "succeed," P1 must start and VA must open. For system B to "succeed," P1 must start and VB must open. The applicable component failure probabilities, which may be considered equivalent to "unavailabilities" for the purposes of this problem, are:

- $u_{p1}$ - Pump 1 fails to start - 0.002
- $u_{VA}$ - Valve A fails to open - 0.0005
- $u_{VB}$ - Valve B fails to open - 0.001

Calculate the probability of each of the four sequences listed on the event tree above. (Potentially useful probability relations and a table of Boolean algebra rules are attached.)
Some probability relations:

1. For independent events $X$ and $Y$ --

   \[ P(X \text{ or } Y) = P(X) + P(Y) - P(X \text{ and } Y) \]

   \[ P(X \text{ and } Y) = P(X) \cdot P(Y) \]

2. For interdependent events $X$ and $Y$ --

   \[ P(X \text{ and } Y) = P(X) \cdot P(Y|X) = P(Y) \cdot P(X|Y), \]

   where \( P(X|Y) \) is the probability that $X$ occurs given that $Y$ has already occurred.

Some Boolean algebra symbols and rules:

* = logical AND (intersection)

+ = logical OR (union)

$X$, $Y$, and $Z$ can represent probabilities, i.e., $X \rightarrow P(X)$, $Y \rightarrow P(Y)$, etc.

\[ X \cdot X = X \]
\[ X + X = X \]
\[ \overline{X} = 1 - X \text{ (the complement of } X) \]
\[ \overline{X} + X = 1 \]
\[ \overline{X} \cdot X = 0 \]
\[ X \cdot (Y + Z) = (X \cdot Y) + (X \cdot Z) \]
\[ X + (Y \cdot Z) = (X + Y) \cdot (X + Z) \]
\[ X \cdot (X + Y) = X \]
\[ X + (X \cdot Y) = X \]