Problem 1 (10 minutes)

The "continuous slowing-down approximation" (CSDA) pretends that neutrons lose energy continuously as they travel through a material, instead of the reality that they lose energy in discrete chunks via discrete scattering events. Under the CSDA, each neutron of a given energy is assumed to gain lethargy (lose energy) at exactly the average rate. This lethargy gain rate is:

\[
\text{lethargy gain per cm} = \xi(E) \frac{\text{lethargy}}{\text{scatter}} \Sigma_s(E) \frac{\text{scatters}}{\text{cm}}.
\]

Suppose that in some material neither \( \xi \) nor \( \Sigma_s \) changes with energy for energy between 1 eV and 100 keV. Under the CSDA, in terms of \( \xi \) and \( \Sigma_s \), what is the total path length that a neutron travels in slowing down from 100 keV to 100 eV?

Problem 2 (10 minutes)

A very small fraction of delayed neutrons (\( \beta = 0.007 \)) can significantly increase the effective neutron lifetime in a reactor with prompt neutron lifetime 10^{-4} seconds. Estimate the corresponding average neutron lifetime given to a single delayed neutron group with decay constant = 0.2 secs^{-1}.

Problem 3 (15 minutes)

Write out the general multigroup diffusion equations for a four-group model, with two thermal groups in which upscattering of neutrons takes place.
Problem 4 (20 minutes)

A thin foil known to contain 0.5 g of isotope $4_{Z}X$ is placed near an operating reactor for 30 minutes, after which it is removed and "cooled" for 2 hours. It is then "counted" using a high-purity germanium detector. During 15 minutes 10,500 counts are registered (after subtracting background) near gamma energy $E_{A+1}$, which is a known characteristic gamma energy from the decay of isotope $^{A+1}_{Z}X$. The half-life of $^{A+1}_{Z}X$ is known to be one hour.

Shortly after the irradiation of the known foil, with the reactor still operating as before, a second foil is placed in the same location as the first and left for 30 minutes, after which it is removed and "cooled" for 1 hour. It is then counted with the same detector, having been placed in exactly the same location as the "known" sample. During 10 minutes 8,000 counts are registered near $E_{A+1}$ (after subtracting background).

a) (50%) What is the mass of isotope $^{4}_{Z}X$ in the second foil? Important: state all assumptions!

b) (50%) Suppose that the second foil was thicker than the first. Would this make your answer to part (a) incorrect? If so, would the correct answer be lower or higher than your answer? Explain.

Problem 5 (20 minutes)

Consider an infinite, source-free slab. Each half of the slab consists of pure absorbing and non-fissioning materials. These materials are significantly different such that the left half of the slab is optically thick and the right one is optically thin, i.e. $\Sigma^{left}_l > l$ and $\Sigma^{right}_l < < l$ where $h$ is the half-width of the slab. A monoenergetic, isotropic angular flux of neutrons is incident on the left boundary. The absorption cross sections ($\Sigma^{left}_a$, $\Sigma^{right}_a$) and the width of the slab ($H=2h$) as well as energy of neutrons ($E_0$) are given.

a) (80%) Determine the spatial distribution of neutron number density in the slab. Present your result for the left half ($0 \leq x \leq h$) and right half ($h < x < 2h$) separately.

b) (10%) Determine the spatial distribution of the partial current density from right to left [$J^-$] in the slab.

c) (10%) Justify the theoretical approach that you used to get your results in part (a) of this problem.
Problem 6 (25 minutes)

A hypothetical material ("nonobtainium") is a strict absorber of neutrons and displays a strong resonance between 6 and 7 MeV that results in an energy dependence, at room temperature, of its absorption cross section given by \( \Sigma_a(E) = 100 \, \text{cm}^{-1} \) for energies \( E \) between 6.45 and 6.55 MeV, and \( \Sigma_a(E) = 1 \, \text{cm}^{-1} \) otherwise.

a) (20 %) Sketch the energy dependence of the absorption cross section of nonobtainium, at room temperature, for energies between 6 and 7 MeV.

b) (20 %) Consider a semi-infinite slab of nonobtainium, extending say from \( x=0 \) to \( x=\infty \), and a normally incident beam of neutrons that is uniformly distributed in energy for energies between 6 and 7 MeV. If the material is at room temperature, calculate the corresponding dependence upon position \( x \) of the group absorption cross section of nonobtainium, for a group extending from 6 to 7 MeV (10 %). Compute numerically the values of this group cross section at the surface of the slab and also at a depth of 0.1 cm (10%).

c) (20%) Briefly discuss the general phenomenon of self-shielding (15%), and use the results of the preceding part to illustrate it (5%).

d) (20%) At operating temperature in a reactor, Doppler broadening modifies the absorption cross section of nonobtainium so that the effective cross section is given by \( \Sigma_a(E) = 50 \, \text{cm}^{-1} \) for energies \( E \) between 6.4 and 6.6 MeV, and \( \Sigma_a(E) = 1 \, \text{cm}^{-1} \) otherwise. Repeat part b), except now for the slab at reactor operating temperature.

e) (20%) Discuss generally how Doppler broadening contributes to the control of a nuclear reactor (15%), and use the result of the preceding part to illustrate this effect (5%).
Problem 7 (20 min)

You are given a large block of beryllium (2m on a side) that contains a small deuterium target at its center, on which 300-keV deuterons from an accelerator impinge. (The deuterium target can be assumed to produce a steady state, point source of 2.5 MeV neutrons. The beam reaches the center of the target through a very tiny hole bored in the beryllium block.)

You have in your possession indium foils (95.7 atom percent In-115) and cadmium covers for the foils. Assume that you have the ability to place the foils (with or without cadmium covers) wherever you wish in the beryllium block. Finally, you have a proportional counter that can detect the beta particles emitted by the decay of In-116 (55 minutes half-life). Describe how you would perform an experiment to determine the Fermi age in beryllium from 2.50 MeV to 1.45 eV (the energy of the large neutron capture resonance of In-115) using the just described equipment. Be sure to state:

1) your experimental procedure,
2) the properties of indium and cadmium, if necessary, that make these materials attractive for measuring the Fermi Age in any moderator to a 1.45 eV,
3) how the Fermi age in beryllium from 2.5 MeV to 1.45 eV can be obtained from the experimentally measured quantities,
4) the relationship of the "measured" age to the expected age in beryllium for fission neutrons to an energy of 1.45 eV.