1. (15 min)\(^{149}\)Sm is a member of the following fission product decay chain:

\[
fission \rightarrow ^{149}\text{Nd} \rightarrow ^{149}\text{Pm} \rightarrow ^{149}\text{Sm}
\]

The half-life of \(^{149}\)Nd is short. Thus, we can neglect the \(^{149}\)Nd and assume that fission reaction yields \(^{149}\)Pm directly. \(^{149}\)Sm is stable and has a large neutron absorption cross section. \(^{149}\)Sm is a strong fission product poison. Assume a homogeneous reactor and one energy group for all neutrons.

a. (10 min) For a reactor operating at a constant flux, \(\phi_0\), derive the expressions for the equilibrium concentrations of \(^{149}\)Pm and \(^{149}\)Sm.

b. (5 min) What is the expression to approximate the corresponding negative reactivity for the equilibrium concentration of \(^{149}\)Sm?

2. (15 min) Consider a bare homogeneous spherical reactor (no reflector) with macroscopic fission cross section \(\Sigma_f\), average recoverable energy release per fission \(E_f\), and extrapolated outer radius \(R\).

a. (5 min) What is the location of the maximum power density in the reactor?

b. (10 min) Derive an expression for the maximum power density divided by the average power density in the reactor.

\[
\text{Hint: } \int_a^b dx \cdot x \cdot \sin(bx) = \frac{1}{b^2} \left[ \sin(ba) - ba \cdot \cos(ba) \right]
\]

3. (15 min) Answer the following questions:

a. (5 min) Write the general form (i.e., including time dependence, energy dependence, and general scattering) of the neutron transport equation in the case of three-dimensional general geometry.

b. (5 min) Write suitable boundary conditions for the neutron transport equation of part (a).

c. (5 min) Write your equation of part (a), and your boundary conditions of part (b), for the special case of steady-state, mono-energetic, isotropically scattering transport.

4. (10 min) Consider a one-region homogeneous (uniform properties) reactor configuration. Give a physical discussion of the difference between the prompt neutron lifetime \(l_p\) and the mean neutron generation time \(\Lambda\) for both infinite and finite reactors. Write both parameters in terms of quantities describing neutron absorption and leakage.
5. (15 min) For a homogeneous slab of width $a$, we measure a multiplication factor of 0.85. For a slab with twice the thickness (that is a slab with width $2a$), the multiplication factor is 0.9. You are given that $a=100$ cm. Determine the infinite medium multiplication factor and the diffusion length for this material. Assume 1-group diffusion theory to be valid.

6. (15 min) You are the reactor manager for a TRIGA research reactor (a small, thermal, pool-type research reactor facility). You are asked to perform an experiment to measure the integral and differential control rod worths [$\rho(z)$ and $d\rho(z)/dz$, respectively] for a single rod in the reactor using the positive period method.
   a. (5 min) Describe how you would perform this experiment.
   b. (5 min) Sketch what you would expect the differential rod worth curve to look like.
   c. (5 min) Sketch what you would expect the integral rod worth curve to look like.

7. (15 min) You perform an experiment to measure the fuel temperature coefficient of reactivity for a small research reactor. You place the reactor at three different steady-state power levels and measure the fuel temperature at these power levels as well as note the reactivity change as indicated by changes in the control rod positions. The data are shown in Table I. Assume that feedback effects from temperature changes in the coolant and other materials are negligible.
   a. (5 min) From the data in Table I, estimate the fuel temperature coefficient of reactivity for the core.
   b. (10 min) Describe how the sign and magnitude of this coefficient of reactivity can affect the safe operation of the reactor.

<table>
<thead>
<tr>
<th>Power (kW)</th>
<th>Fuel Temperature (°F)</th>
<th>Total Rod Worth (cents)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>90.2</td>
<td>-905</td>
</tr>
<tr>
<td>200</td>
<td>349.5</td>
<td>-965</td>
</tr>
<tr>
<td>1000</td>
<td>694.9</td>
<td>-1045</td>
</tr>
</tbody>
</table>

TABLE I
Measured Data for Fuel Temperature Coefficient of Reactivity Experiment
8. (20 min) Consider the following 1-D reactor geometry. The system is either critical or supercritical.

![Diagram of reactor geometry]

We wish to calculate the $k$-eigenvalue and eigenfunction for this system assuming one-group diffusion theory with an extrapolation distance of $\frac{2}{3\Sigma_t}$. The fuel has the following macroscopic cross sections: $\Sigma_{t1}$, $\Sigma_{a1}$, and $\Sigma_{f1}$. The moderator has the following macroscopic cross sections: $\Sigma_{t2}$ and $\Sigma_{a2}$.

a. (4 min) Give the diffusion equation satisfied by the scalar flux, $\phi_1$, in the fuel region.

b. (4 min) Give the diffusion equation satisfied by the scalar flux, $\phi_2$, in the moderator region.

c. (4 min) Assume a scalar flux solution of the form $\phi_1 = \cos(\alpha x)$ in the fuel and derive an expression for $\alpha$. Note that $\alpha$ is a function of certain cross sections and $k$.

d. (4 min) Assume a scalar flux solution of the form $\phi_2 = a \exp(\beta x) + b \exp(-\beta x)$ in the moderator where $a$ and $b$ are constants. Derive an expression for $\beta$. Note that $\beta$ is a function of certain cross sections.

e. (4 min) The eigenfunction solution that we have given solves the diffusion equation in each region. Give the three additional equations required to determine $k$, $a$, and $b$. 