(1) \textbf{(15 min)} The diffusion approximation in 1-D slab geometry is consistent with the following approximation for the angular flux (whose units are particles/(cm$^2$-s-ster)):

$$
\psi(x, \mu) = \phi(x) + \frac{3\mu J(x)}{4\pi},
$$

where $\phi(x)$ is the scalar flux and $J(x)$ is the net current density.

(a) \textbf{(10 min)} Use this approximation together with Fick’s law to derive the “Mark” vacuum boundary condition at the left face of a slab:

$$
\phi(x_L) - \frac{1}{\sqrt{3}} \lambda_t \left[ \frac{d\phi}{dx} \right]_{x=x_L} = 0,
$$

where $x = x_L$ is the position of the left boundary and $\lambda_t$ is the total mean-free path. (The Mark boundary condition expresses that the incident angular flux is zero in one particular direction.)

(b) \textbf{(5 min)} Give a graphical interpretation of this boundary condition in terms of an extrapolation of the solution at the boundary.

(2) \textbf{(15 min)} Consider a low-power subcritical reactor with a Pu-Be neutron source. Control rods are partially removed, inserting a certain amount of reactivity. After some time the neutron flux reaches a stable value that is double its original value. Now control rods are moved again so that exactly the same amount of reactivity is again added. What will happen to the neutron flux in the short term and long term?

(3) \textbf{(15 min)} Consider a thin foil of material composed of a single nuclide, subjected to a constant neutron flux of $\phi$ (neutrons per unit area per unit time). The foil volume is $V$ and its neutron-capture cross section averaged over the neutron spectrum is $\Sigma_\gamma$. When the original nuclide captures a neutron, the product nucleus is radioactive.

(a) \textbf{(5 min)} Provide an expression for the “saturation activity,” $A_{\text{sat}}$, of the foil in this flux.

(b) \textbf{(5 min)} Derive an expression for the foil activity, $A(t)$, as a function of irradiation time $t$, product-nucleus decay constant $\lambda$, and saturation activity $A_{\text{sat}}$.

(c) \textbf{(5 min)} Determine how much irradiation time is needed to induce an activity of 10% of the saturation activity. Express your answer in terms of the nuclide’s half-life.
(4) (15 min) Assume one delayed-neutron precursor group. Sketch graphs of the neutron population, \( n(t) \), and delayed-neutron precursor population, \( C(t) \), for a thermal reactor that is subjected to the following reactivity insertions after operating for a long time at steady state:

(a) (7.5 min) Positive reactivity less than 1$ (\rho < \beta)$;

(b) (7.5 min) Negative reactivity of magnitude greater than 1$ (\rho < -\beta)$.

(5) (25 min) Consider three-group diffusion theory for a bare homogeneous reactor.

(a) (5 min) Assume that essentially all fission neutrons are born in group 1 (the highest-energy group) and group-to-group upscattering is negligible. Write the 3-group diffusion \( k \)-eigenvalue equations for this case.

(b) (10 min) Derive the group-flux ratios \( \phi_2/\phi_1 \) and \( \phi_3/\phi_1 \). Make and state reasonable assumptions that simplify the problem. If you introduce new terms or quantities, define them.

(c) (10 min) Suppose the ratios \( \phi_2/\phi_1 \) and \( \phi_3/\phi_1 \) are given. In terms of these ratios, derive the 3-group-diffusion expression for the multiplication factor of a bare homogeneous reactor.

(6) (25 min) For the questions below, use the following for power coefficient of reactivity \( \alpha_P \) and feedback from equilibrium \(^{135}\text{Xe} \) at reactor power \( P \):

\[
\alpha_P = -0.5 \text{ $/GW}$
\]

\[
\rho_{^{135}\text{Xe}}^\text{eq}(P) = [-5$] \frac{2P}{1 \text{ $/GW} + 2P}
\]

In each part below, briefly explain your reasoning. You will not receive credit for an answer without an explanation. Make the following simplifying assumptions:

- Ignore reactivity feedback from nuclides other than \(^{135}\text{Xe} \).
- Ignore decay heat, so \( P(t) \) is proportional to fission rate at time \( t \).

(a) (5 min) A source-free reactor operates at 3.0 GW for more than a week. At this time control rods are moved into the core, inserting reactivity of \(-1$\). Let \( P_a \) be the power level several minutes later, and write down an algebraic equation in which \( P_a \) is the only unknown. Explain.

(b) (5 min) If the operators take no action, a few hours later is the power (call it \( P_b \)) greater than, less than, or equal to \( P_a \)? Explain.

(c) (5 min) A few days later what is the power? Call it \( P_c \) and write an algebraic equation in which \( P_c \) is the only unknown. Explain.

(d) (10 min) The operators now move the control rods back to their original positions. Describe any power increases and decreases during the next few days, and quantify the final steady-state power level, \( P_{final} \). (Quantification could take the form of an algebraic equation in which \( P_{final} \) is the only unknown.)
Consider the angular flux, $\psi(\vec{r}, E, \Omega)$, which has units of particles per unit (area-time-energy-steradian). Consider the corresponding scalar flux, $\phi(\vec{r}, E)$, which has units of particles per unit (area-time-energy-steradian).

(a) (5 min) What is the fundamental equation that relates the scalar flux to the angular flux?

(b) (5 min) Assume that the angular flux is isotropic and express it in terms of the scalar flux.